Foundations of Analysis / MAT 3213.001

Midterm 3 / April 22, 1998 / Instructor: D. Gokhman

Name: ____

1. (40 pts.) Suppose A and B are subsets of $\mathbf{R}$. For each the following statements determine whether the statement is true and prove the statement or give a counterexample.
   (a) $\overline{A \cap B} = \overline{A} \cap \overline{B}$
   (b) $\overline{A \cup B} = \overline{A} \cup \overline{B}$
   (c) If A and B are dense in $\mathbf{R}$, then so is $A \cap B$.
   (d) If A and B are dense in $\mathbf{R}$, then so is $A \cup B$.

2. (20 pts.) Suppose $A \subseteq \mathbf{R}$.
   (a) Show that the interior of A is empty if, and only if, $\mathbf{R} \setminus A$ is dense in $\mathbf{R}$.
   (b) Show that if A is countable, then $\mathbf{R} \setminus A$ is dense in $\mathbf{R}$.
   (Hint: Show that if the interior of A is not empty, then A contains an interval.)

3. (20 pts.) Suppose $A \subseteq \mathbf{R}$ is not empty and bounded above, but $\max A$ does not exist. Show that A has a limit point in $\mathbf{R}$.

1	2(a-b)	2(c-d)	4	total (80)	%