Foundations of Analysis / MAT 3213.001

Midterm 2 / March 30, 1998 / Instructor: D. Gokhman

Name: ______________________________

1. (10 pts.) Suppose $f\colon A \to B$ and $g\colon B \to C$.
   Prove that if $g$ is 1-1 and $g \circ f$ is onto, then $f$ is onto.

2. (20 pts.) Suppose $f\colon A \to B$ is onto. Prove or give a counterexample to each of the following statements.
   (a) If $A$ is countable, then $B$ is countable.
   (b) If $B$ is countable, then $A$ is countable.

3. (20 pts.) Suppose $A$ is a nonempty bounded subset of $\mathbf{R}$.
   (a) Prove that $\sup A$ is not an interior point of $\mathbf{R} \setminus A$.
   (b) Prove that if $A$ is closed, then $\sup A \in A$.

4. (25 pts.) Prove or disprove that $A$ is a closed subset of $\mathbf{R}$, if
   (a) $A$ is a singleton
   (b) $A$ is finite
   (c) $A = \mathbf{Z}$
   (d) $A = \mathbf{Q}$
   (e) $A = (-\infty, 0]$
   (f) Extra credit: $A = \left\{ \frac{1}{n} \colon n \in \mathbf{Z}^+ \right\} \cup \{0\}$

| 1 | 2 | 3 | 4 | total (75) | % |
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