

Name: _____

Show all work.

1. Prove by induction that $\sum_{k=1}^n \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$ for $n = 1, 2, \dots$
2. Prove by induction that $5^n \geq 1 + 4^n$ for $n = 1, 2, \dots$
3. Let $A = \{1, 2, 3\}$ and let $R = \{[1, 1], [2, 3], [3, 2]\}$ be a relation on A . Find $R \circ R$ and $R \circ R \circ R$ and sketch a digraph for each of the relations $R, R \circ R, R \circ R \circ R$
4. Define a relation R on $\mathbf{R} \times \mathbf{R}$ by $[x, y]R[r, s] \Leftrightarrow x - y = r - s$. Prove that R is an equivalence relation. On the same set of axes sketch the equivalence class of $[2, 2]$ and the equivalence class of $[2, 3]$
5. Explain why the set of all even integers $2\mathbf{Z}$ and the set of all odd integers $1 + 2\mathbf{Z}$ form a partition of \mathbf{Z} . Describe the equivalence relation on \mathbf{Z} whose quotient set is the above partition $\{2\mathbf{Z}, 1 + 2\mathbf{Z}\}$

1	2	3	4	5	total (50)