Midterm 2 / 2023.3.31 / MAT 3013.001 / Foundations of Mathematics

Name: ____

Please show all work.

1. Let x_n be the sequence of integers recursively defined by

$$x_0 = 0$$
$$x_1 = -5$$
$$x_n = 7x_{n-1} - 6x_{n-2} \text{ for } n > 1$$

   Prove by induction on n that $x_n = 1 - 6^n$ for all $n \geq 0$

2. For each natural number $n > 0$ let A_n be the interval $(-\frac{1}{n}, 0)$
   (a) Find the union $\bigcup_{n=1}^{\infty} A_n$ and the intersection $\bigcap_{n=1}^{\infty} A_n$ of this family of sets.
   (b) Prove your assertions.

3. Define a relation S on the real line $\mathbf{R}$ by $aSb \Leftrightarrow a - b$ is an integer multiple of 2π
   (a) Prove that S is an equivalence relation.
   (b) Describe the equivalence classes.
   (c) Extra credit: Explain why the quotient set $\mathbf{R}/S$ (the set of all equivalence classes) is in one-to-one correspondence with the unit circle.

4. For each of the following relations S on $\mathbf{R}$, determine whether S is reflexive, whether S is symmetric, and whether S is transitive. Explain.
   (a) $aSb \Leftrightarrow ab = 1$
   (b) $aSb \Leftrightarrow ab \geq 0$

1	2	3	4	total (40)