Midterm 1 / 2023.2.24 / MAT 3013.001 / Foundations of Mathematics

Name: \_

Please show all work.

- 1. If P, Q, R are propositions, use a truth table to prove that  $(P \land Q) \lor R \Leftrightarrow (P \lor R) \land (Q \lor R)$
- 2. Using formal language and appropriate quantifiers, translate into symbolic form the following sentences. Determine whether they are equivalent and explain why or why not.
  - Every integer is even or odd.
  - Every integer is even or every integer is odd.
- 3. An integer greater than 1 is called prime precisely when its only positive divisors are itself and 1. Write out this statement in the language of formal logic using appropriate quantifiers. Then negate it (and simplify) and write out the negation in words.
- 4. Suppose A, B, C are sets. For each of the following statements determine whether it is true. If true, prove it. If not, provide a concrete counterexample and explain why it works.
  - (a)  $(A \subseteq B \land B \subseteq C) \Rightarrow A \subseteq C$
  - (b)  $(A \subseteq B \land B \not\subseteq C) \Rightarrow A \not\subseteq C$

1	2	3	4	total (40)