Midterm 1 / 2020.2.26 / MAT 3013.001 / Foundations of Mathematics

Name: ____________________________________________

Please show all work.

1. Construct a truth table to establish the equivalence of implication with its contrapositive. In other words, use a truth table to prove $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$.

2. Translate "everybody loves somebody sometime" into the formal language of predicate calculus. Negate it and translate the negation back into human language.

   Hint: Let $p(x, y, t)$ denote "$x$ loves $y$ at time $t$."

3. Show that for arbitrary sets $A, B, C, D$ we have $(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$ and provide a concrete counterexample to subset the other way.

4. For each $n \in \mathbf{N}$ let $A_n = \{x \in \mathbf{R}\colon 0 \leq x \leq 1/n\} = [0, 1/n]$. Find the union and the intersection of this family of sets. Prove your assertions.

5. Use the principle of mathematical induction to prove Faulhaber's formula

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

| 1 | 2 | 3 | 4 | 5 | total (50) |
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