Final exam / 2020.5.11 / MAT 3013.001 / Foundations of Mathematics

Name: ____________________

Please show all work.

1. Let $f\colon \mathbf{R}\setminus\{-1\}\to\mathbf{R}$, $f(x)=\dfrac{x}{x+1}$.

   (a) Prove that f is not an increasing function on its domain, but its restrictions to intervals $f|_{(-\infty,-1)}$ and $f|_{(-1,\infty)}$ are strictly increasing.

   (b) Find a codomain for $f|_{(-1,\infty)}$ that makes the function bijective. Find the compositional inverse of our function. Sketch both our function and its inverse on the same set of axes.

2. Let $f\colon \mathbf{R}\to\mathbf{R}$, $f(x)=x^2+1$. Find and sketch:

   (a) $f([-1,0]\cup[2,4])$.

   (b) $f^{-1}([-1,5]\cup[17,26])$.

3. Suppose $f\colon A\to B$ is a function and R is a relation on A given by $xRy \Leftrightarrow f(x)=f(y)$.

   (a) Prove that R is an equivalence relation.

   (b) Prove that nonempty fibers of f are equivalence classes under R and *vice versa*.

4. Suppose $f\colon A\to B$ is a function and R is an equivalence relation on B with exactly two distinct equivalence classes $U,V\subseteq B$. Prove that $\{f^{-1}(U), f^{-1}(V)\}$ is a partition of A.

1	2	3	4	total (40)