Midterm 1 / 2015.10.1 / MAT 3013.002 / Foundations of Mathematics

Name: _____

Please show all work.

1. Construct the truth table for $(p \to q) \vee (q \to p)$ and draw a conclusion.

2. Write out a formal definition for divisibility. Start like this: integer m divides integer n (written $m|n$) means ... Hint: n should be m times something. Negate the definition to obtain a formal description for $m \not| n$.

3. Write out a formal definition of a prime number. Start like this: a natural number n is prime means ... Hint: n must be at least 2 and the only natural numbers that divide n are 1 and n (you may use the $m|n$ and $m \not| n$ shorthand to deal with divisibility). Negate the definition to obtain a formal description for n being not prime.

4. Prove or disprove subset in each direction between $(A \cap C) \times (B \cap D)$ and $(A \times B) \cap (C \times D)$.

5. Suppose $f : \mathbf{R} \to \mathbf{R}$ is given by $f(x) = \dfrac{1}{1+x^2}$. Find the following.

   (a) $f_*(\mathbf{R})$ (b) $f^*(\mathbf{R})$ (c) $f^*(\{\frac{1}{2}\})$ (d) $A \subseteq \mathbf{R}$ such that $A \neq \varnothing \wedge f^*(A) = \varnothing$

6. Construct functions f and g such that $g \circ f$ is bijective, but f is not surjective and g is not injective. Be sure to specify domains and co-domains. Extra credit: can you do this problem with the additional requirement that all the domains and co-domains be $\mathbf{R}$?

1	2	3	4	5	6	total (60)	%

Prelim. course grade: %