Midterm 1 / 2012.10.18 / MAT 3013.001 / Foundations of Mathematics

Name: _____

Please show all work.

1. Prove by induction that $n! \leq n^n$ for all natural numbers $n \geq 1$.
2. Define a sequence recursively as follows: $a_0 = 0, a_1 = 1$, and $a_n = 5a_{n-1} - 6a_{n-2}$ for all $n \geq 2$. Prove that $a_n = 3^n - 2^n$ for all natural numbers.
3. Construct a truth table for $(p \to q) \leftrightarrow (q \to p)$. Is it a tautology, contradiction, or neither?
4. Prove that integer multiples of 3 is a set. You may assume $\mathbf{Z}$ is a set.
5. Prove or disprove subset in each direction between $(A \cup C) \times (B \cup D)$ and $(A \times B) \cup (C \times D)$.
6. Suppose $f : \mathbf{R} \to \mathbf{R}$ is the principal branch of arctangent. Find the following.
   (a) $f_*(\mathbf{R})$ (b) $f^*((-\infty, -\frac{\pi}{4}])$ (c) $A \subseteq \mathbf{R}$ such that $A \neq \varnothing \wedge f^*(A) = \varnothing$

1	2	3	4	5	6	total (60)	%

Prelim. course grade: %