Name: \_\_\_\_

Please show all work.

- 1. Prove by induction that  $n! \ge 2^n$  for all natural numbers  $n \ge 4$ .
- 2. Prove that  $(p \to (q \land \neg q)) \to \neg p$  is a tautology.
- 3. Negate the statement  $(\exists y)[p(y) \land (\forall x)[\neg q(x) \rightarrow r(x)]].$  Simplify.
- Prove that any interval in R is a set. You may assume R is a set. Hint: you may need to look at some cases.
- 5. Prove or disprove  $(\forall A)[B \in \mathscr{P}(A)] \Rightarrow B = \varnothing$ .
- 6. Prove or disprove  $(A \cup C) \times (B \cup D) \subseteq (A \times B) \cup (C \times D)$ .

| 1 | 2 | 3 | 4 | 5 | 6 | total (60) | % |
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Prelim. course grade: %