

Name: \_\_\_\_\_

Please show all work. Check your answers! 😊

1. Let  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{bmatrix}$ .

- Find a basis for the kernel of  $A$ .
- Find a basis for the image of  $A$ .
- Describe and sketch the kernel and the image of  $A$ .

2. Let  $A = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$ ,  $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ ,  $v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ .

- Explain why  $u$  and  $v$  form a basis for the plane  $\mathbf{R}^2$ .
- Express  $Au$  as a linear combination of  $u$  and  $v$ . Same for  $Av$ .
- What matrix represents the linear map  $x \mapsto Ax$  with respect to the  $u, v$  basis?

3. Let  $P$  be the vector space of polynomials and let  $c > 0$ . Explain why the following subsets  $H$  of  $P$  are subspaces of  $P$ .

- $H = \{p(t) : p(c) = 0\}$ .
- $H = \left\{ p(t) : \int_0^c p(t) dt = 0 \right\}$ .

| 1 | 2 | 3 | total (30) |
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