Midterm 2 / 2016.10.28 / MAT 2233.002 / Linear Algebra

Name: ______________________________

Please show all work.

1. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$.

   (a) Find a basis for the kernel of $A$.

   (b) Find a basis for the image of $A$ and sketch it.

2. Let $A = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Express $A\mathbf{v}_1$ and $A\mathbf{v}_2$ as linear combinations of $\mathbf{v}_1$ and $\mathbf{v}_2$. What matrix represents the linear map $\mathbf{x} \mapsto A\mathbf{x}$ relative to the basis $[\mathbf{v}_1, \mathbf{v}_2]$?

3. Let $P_n$ be the vector space of all real polynomials $p(t)$ with degree $\leq n$ and let $Z_n$ be the set of all $p(t)$ in $P_n$ such that $p(0) = 0$. Prove that

   (a) $Z_n$ is a vector subspace of $P_n$.

   (b) $T$ defined by $T(p(t)) = \int_0^t p(s)\,ds$ is a linear map from $P_n$ to $Z_{n+1}$.

4. (c) Find the matrix for $T$ with respect to bases $[1, t, t^2]$ for $P_2$ and $[t, t^2, t^3]$ for $Z_3$.

   (d) Prove that $T$ is invertible. Find a formula and the matrix for $T^{-1}$.

| 1 | 2 | 3 | 4 | total (40) |
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