Midterm 1 / 2016.9.30 / MAT 2233.002 / Linear Algebra

Name: ____

Please show all work. Check your answers! ◡̈

1. Consider the linear system $\begin{bmatrix} 2x+4y+6z=0 \\ 4x+5y+6z=3 \\ 7x+8y+9z=6 \end{bmatrix}$.

    (a) Use Gauss-Jordan elimination to find all solutions. Show steps. Describe and sketch the solution set.

    (b) Can you expect some solutions to this system for arbitrary right-hand-sides?

2. Suppose $T\colon \mathbf{R}^2 \to \mathbf{R}$ is a linear map and we know its values at some two (column) vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbf{R}^2$ that not scalar multiples of one another: $T(\mathbf{u}) = a, T(\mathbf{v}) = b$.

    (a) Let $S = [\mathbf{u}, \mathbf{v}]$. Explain why S is an invertible matrix. What is rref(S)?

    (b) Let A be the matrix that represents T. Let $B = [a, b]$. Explain why $AS = B$.

    Hint: Since $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x}$ in $\mathbf{R}^2$, $A = [A\mathbf{e}_1, A\mathbf{e}_2] = [T(\mathbf{e}_1), T(\mathbf{e}_2)]$, so compute $AS\mathbf{e}_i = T(S\mathbf{e}_i) = \ldots$.

3. Preceding problem continued:

    (c) Use (a) to solve the matrix equation in (b) for A.

    (d) If $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $a = 1$ and $b = -2$ use your solution in (c) to find A.

4. In each part enter a real 2×2 nonzero nonidentity matrix A such that the linear map $\mathbf{x} \mapsto A\mathbf{x}$ is as given.

    $\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$ (a) orthogonal reflection with respect to a line

    $\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$ (b) orthogonal projection to a line

    $\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$ (c) isotropic dilation

    $\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$ (d) rotation

    $\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$ (e) horizontal shear

1	2	3	4	total (40)