Midterm 2 / 2016.3.25 / MAT 2233.002 / Linear Algebra

Name: ______________________________

Please show all work.

1. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$. Find a basis for the kernel of A. What is the image of A?

2. Let P_3 be the space of all real polynomials $p(t)$ with degree ≤ 3 and let $T\colon P_3 \to P_3$ be the linear map given by $T(p) = p'' - kp$, where k is a constant. Find the matrix that represents T with respect to the basis $[1, t, t^2, t^3]$.

3. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Express $A\mathbf{v}_1$ and $A\mathbf{v}_2$ as linear combinations of $\mathbf{v}_1$ and $\mathbf{v}_2$. What matrix represents the linear map $\mathbf{x} \mapsto A\mathbf{x}$ relative to the basis $[\mathbf{v}_1, \mathbf{v}_2]$?

4. Prove that $\mathbf{v}_1 = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ form a basis for the plane $x + 2y + 3z = 0$. Let M be the matrix with columns $\mathbf{v}_1$ and $\mathbf{v}_2$. Find the QR factorization of M.

1	2	3	4	total (40)