Midterm 1 / 2016.2.19 / MAT 2233.002 / Linear Algebra

Name: _____

Please show all work.

1. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.
   (a) Find all solutions to $Ax = b$. Sketch/describe the solution set.
   (b) Can you expect some solutions to $Ax = b$ for any b? Explain.
2. Suppose A is a 3×2 matrix with trivial kernel.
   (a) Given a vector b, what are all the possibilities for the number of solutions to $Ax = b$? Explain.
   (b) Give concrete examples of A and b that match your predictions above.
3. Let $T: \mathbf{R}^2 \to \mathbf{R}^2$ be orthogonal projection to the main diagonal (graph of $y = x$) in $\mathbf{R}^2$.
   (a) Find a matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x}$ in $\mathbf{R}^2$.
   (b) Express the kernel and the image of T as spans of vectors. Sketch.
4. Suppose $T: \mathbf{R}^2 \to \mathbf{R}$ is a linear map. Let $a = T(2\mathbf{e}_1 + 3\mathbf{e}_2), b = T(3\mathbf{e}_1 + 4\mathbf{e}_2)$, where $\mathbf{e}_1, \mathbf{e}_2$ is the standard basis for $\mathbf{R}^2$. Find a matrix A, in terms of a and b, such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x}$ in $\mathbf{R}^2$.

1	2	3	4	total (40)