Final exam / 2002.12.10 / Linear Algebra / MAT 2233.001

Name: ______________________________

Please show all work and justify your answers.

1. (10 pts.) Let $A = \begin{bmatrix} 3 & -1 & 4 \\ -1 & 2 & -2 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$.

   (a) Find all solutions to $Ax = b$ (b) Describe and sketch the solution set.

2. (10 pts.) Let $T\colon \mathbf{R}^3 \to \mathbf{R}^3$ be the orthogonal projection to the plane $x_1 = 0$ followed by a reflection with respect to the main diagonal $x_2 = x_3$ of that plane.

   (a) Find the matrix for T with respect to the standard basis of $\mathbf{R}^3$.
   (b) Describe and sketch the image and the kernel of T.

3. (10 pts.) Suppose $A \neq 0$. What are all the possibilities for the number of solutions to the linear system $Ax = b$ if A is 3×1? If A is 1×1? If A is 1×3?

4. (10 pts.) Suppose T is a linear map such that $T \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \\ 3 \end{bmatrix}$ and $T \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 2 \end{bmatrix}$.

   (a) Find the matrix for T with respect to the standard bases of $\mathbf{R}^2$ and $\mathbf{R}^3$.
   (b) Is T 1-1? Onto? Explain.

5. (10 pts.) Determine whether the following are vector subspaces of $\mathbf{R}^n$. If yes, explain. If no, provide a concrete counterexample.

   (a) The solution set of a consistent system $Ax = b$
   (b) The image of A

6. (10 pts.) Determine whether the matrices $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ and $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ are similar.

7. (10 pts.) Let L be the plane in $\mathbf{R}^3$ given by $2x_1 - x_2 + 3x_3 = 0$ and let $v = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$.

   Define a linear transformation T of L by $Tx = v \times x$. Find a basis for L and the matrix of T with respect to this basis. What is the determinant of T?

8. (10 pts.) Find all eigenvalues and the corresponding eigenvectors of

   (a) $A = \begin{bmatrix} 5 & 2 \\ 4 & -2 \end{bmatrix}$ (b) Orthogonal projection of the plane to the line $2x_1 - x_2 = 0$

9. (10 pts.) Suppose $A = \begin{bmatrix} 2 & -3 \\ -4 & -9 \end{bmatrix}$. It can be shown that A has eigenvalues -10 and 3 with corresponding eigenvectors $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

   Solve the dynamical system $x(n) = Ax(n-1)$ with initial condition $x(0) = \begin{bmatrix} 11 \\ 5 \end{bmatrix}$.

1	2	3	4	5	6	7	8	9	total (90)
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