Linear Algebra / MAT 2233.001
Midterm 2 / 2002.4.24 / Instructor: D. Gokhman
Name: $\qquad$

1. (10 pts.) Find bases for the kernel and the image of $A=\left[\begin{array}{llll}1 & 3 & 1 & 2 \\ 1 & 3 & 1 & 4\end{array}\right]$
2. ( 10 pts.) Let $v=(1,2,3)$ and define $T: \mathbf{R}^{3} \rightarrow \mathbf{R}^{3}$ by $T(x)=v \times x$. Find bases for the kernel and the image of $T$.
3. ( 10 pts .) Find two subspaces of $\mathbf{R}^{2}$ whose union is not a subspace of $\mathbf{R}^{2}$.
4. (10 pts.) Suppose $v$ and $w$ are linearly independent vectors in $\mathbf{R}^{3}$. Define $T: \mathbf{R}^{3} \rightarrow \mathbf{R}$ by letting $T(x)=\operatorname{det}\left[\begin{array}{lll}x & v & w\end{array}\right]$. Describe the kernel of $T$. What is its dimension?
5. (20 pts.) Let $A=\left[\begin{array}{ll}2 & 1 \\ 2 & 3\end{array}\right]$.
(a) Find all eigenvalues of $A$ and the corresponding eigenvectors.
(b) Find a formula for $A^{n}$.

| 1 | 2 | 3 | 4 | 5 | total (60) | $\%$ |
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