Linear Algebra / MAT 2233.001
Midterm 1 / March 6, 2002 / Instructor: D. Gokhman

Name: $\qquad$

1. (10 pts.) Let $A=\left[\begin{array}{lll}1 & 3 & 0 \\ 0 & 0 & 1\end{array}\right]$ and $b=\left[\begin{array}{l}1 \\ 2\end{array}\right]$. Find all solutions to $A x=b$. Describe and sketch the solution set.
2. (10 pts.) Let $T: \mathbf{R}^{2} \rightarrow \mathbf{R}^{2}$ be the orthogonal projection to the line $x=2 y$. Find the matrix $A$ such that $T(x)=A x$ for all $x$.
3. (10 pts.) Give an example of a $3 \times 2$ matrix $A$ and a vector $b$ such that $A x=b$ has a unique solution.
4. (10 pts.) Suppose $A$ is a $3 \times 2$ matrix and $A x=0$ has the unique zero solution. What can you say about the number of solutions of $A x=b$ for an arbitrary vector $b ?$
5. (10 pts.) Find all linear maps $T: \mathbf{R}^{2} \rightarrow \mathbf{R}^{2}$ such that $T\left[\begin{array}{l}1 \\ 1\end{array}\right]=\left[\begin{array}{l}2 \\ 1\end{array}\right]$ and $T\left[\begin{array}{r}1 \\ -1\end{array}\right]=\left[\begin{array}{l}1 \\ 2\end{array}\right]$.

| 1 | 2 | 3 | 4 | 5 | total (50) | $\%$ |
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