Linear Algebra / MAT 2233.001
Final exam / 2002.5.10 / Instructor: D. Gokhman

Name: ______________________________

1. (10 pts.) Let $A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. Find all solutions to $Ax = b$. Describe and sketch the solution set.

2. (10 pts.) Let $T\colon \mathbf{R}^2 \to \mathbf{R}^2$ be the reflection with respect to the line $x = \sqrt{3}\,y$. Find the matrix A such that $T(x) = Ax$ for all x. Hint: you should be able to recognize the angle of inclination of the line.

3. (10 pts.) Give an example of a 3×2 matrix A and vectors u and v such that $Ax = u$ has a unique solution while $Ax = v$ has no solutions.

4. (10 pts.) Suppose A is a 3×2 matrix and $Ax = 0$ has many solutions. What can you say about the number of solutions of $Ax = b$ for an arbitrary vector b?

5. (10 pts.) Find all linear maps $T\colon \mathbf{R}^2 \to \mathbf{R}^2$ such that $T\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $T\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

6. (10 pts.) Suppose $T\colon \mathbf{R}^3 \to \mathbf{R}^3$ is the orthogonal projection to the plane $x + 2y + 3z = 0$. Find bases for the kernel and the image of T.

7. (10 pts.) Explain why the intersection of two subspaces of $\mathbf{R}^n$ is a subspace of $\mathbf{R}^n$.

8. (10 pts.) Suppose A is a 3×3 matrix with rows u, v, w and $\det A = 5$. Let B be 3×3 matrix with rows u, $u + v$, $u + v + 2w$. Use properties of determinant to find $\det B$.

9. (20 pts.) Let $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.

   (a) Find all eigenvalues of A and the corresponding eigenvectors.
   (b) Find a formula for A^n.

1	2	3	4	5	6	7	8	9	total (100)	%