Linear Algebra / MAT 2233
Midterm 2 / December 1, 1999 / Instructor: D. Gokhman

Name: ______________________________________

Please show all work and box the answers, where appropriate.

1. (10 pts.) Determine whether each of the following subsets is a vector subspace. If yes, express it as a span or a null space. If not, explain why not by giving an explicit example of how an axiom fails.

   (a) Polynomials of the form $a + bt^2$ in the space of all polynomials in t.

   (b) $\left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : \begin{array}{c} a - 2b + c = 0 \\ \text{and } b - c = 0 \end{array} \right\}$ in $\mathbf{R}^3$ (c) $\left\{ \begin{bmatrix} s+1 \\ t \\ s \end{bmatrix} : s, t \text{ in } \mathbf{R} \right\}$ in $\mathbf{R}^3$

2. (10 pts.) Find a basis for: (a) null $\begin{bmatrix} -1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$, (b) col $\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

   Sketch both spaces.

3. (10 pts.) Let $\mathcal{B} = \{1 + t, t\}$ be a basis for $\mathbf{P}_1$ and let $p = t - 1$. Find $[p]_{\mathcal{B}}$.

4. (10 pts.) Is the sequence $\{(t + 1)^2, t^2 + 1, t^2\}$ linearly independent in $\mathbf{P}_2$?

5. (10 pts.) Let $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$.

   (a) Find the characteristic polynomial of A.

   (b) Find the spectrum (the set of all eigenvalues) of A.

   (c) Find eigenvectors corresponding to each eigenvalue of A.

   (d) Find an invertible 2×2 matrix P and a diagonal 2×2 matrix D such that $A = PDP^{-1}$.

   (e) Compute P^{-1} and verify that $A = PDP^{-1}$.

1	2	3	4	5	total (50)	%