Linear Algebra / MAT 2233

Midterm 1 / October 20, 1999 / Instructor: D. Gokhman

Name: ______________________________

Please show all work and box the answers, where appropriate.

1. (10 pts.) Take an augmented matrix $A = \begin{bmatrix} 2 & -2 & 4 & 0 \\ 1 & 1 & -2 & 2 \end{bmatrix}$.
   (a) Use the row reduction algorithm to bring A to echelon form. Then to reduced echelon form.
   (b) Answer the fundamental questions on existence and uniqueness of solutions of the corresponding system.
   (c) Find all solutions.
   (d) Sketch and describe the solution set.

2. (10 pts.) Let $u = (1, -1, 1)$, $v = (2, 0, 1)$, $w = (3, 1, 1)$.
   (a) Sketch span $\{u, v\}$.
   (b) Is w in span $\{u, v\}$?
   (c) Same with $w = (3, 2, 1)$.

3. (10 pts.) For the following sequences of vectors:
   (a) Sketch and describe the span of the sequence (except in (vi)).
   (b) Determine whether the sequence is linearly independent. Show work or explain.

   (i) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ -5 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -6 \end{bmatrix}$

4. (10 pts.) Suppose $f \colon \mathbf{R}^2 \to \mathbf{R}^3$ is linear, $f\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$, $f\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix}$.
   (a) Find $f\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$ and $f\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$.
   (b) Find the matrix that represents f with respect to the standard basis.
   (c) Is f 1-1? Is f onto? Explain.
   (d) Sketch and describe the range of f.

5. (10 pts.)
   (a) Suppose A, B, C, I are $n \times n$ invertible matrices and I is the identity matrix. Solve matrix equation $A(CX - I)B = ABC$ for the $n \times n$ matrix X. Simplify.
   (b) Determine whether $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ is invertible, and if so, find the inverse.

1	2	3	4	5	total (50)	%