Linear Algebra / MAT 2233.901
Midterm 2 / April 29, 1999 / Instructor: D. Gokhman

Name: ____________________

Please show all work.

1. (10 pts.) Suppose A, B, C are invertible $n \times n$ matrices.
Find an $n \times n$ matrix X such that $AB(C - X)B = I$.

2. (20 pts.) Let $A = \begin{bmatrix} 1 & 2 & 2 & -1 \\ -1 & -2 & 0 & -1 \\ 3 & 6 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.
It can be checked that A is row equivalent to B.
Find bases for $\operatorname{nul} A$ and $\operatorname{col} A$. Describe and sketch $\operatorname{col} A$.

3. (30 pts.) Determine whether each of the following sets is a subspace of Euclidean space and give reasons for your answer. Sketch each set.

(a) $\left\{ \begin{bmatrix} s - t \\ t \end{bmatrix} : s, t \text{ in } \mathbf{R} \right\}$ (b) $\left\{ \begin{bmatrix} s \\ t - 1 \\ s + t \end{bmatrix} : s, t \text{ in } \mathbf{R} \right\}$ (c) $\left\{ \begin{bmatrix} s + t + 1 \\ t - 1 \\ s + 2 \end{bmatrix} : s, t \text{ in } \mathbf{R} \right\}$

4. (20 pts.) Find $[v]_{\mathcal{B}}$, where

(a) $v = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$ in $\mathbf{R}^2$ and $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$.

(b) $v = 3 + t + 2t^2$ in P_2 and $\mathcal{B} = \{1 - t^2, 1 - t, 1 + t^2\}$.

1	2	3	4	total (80)	%