Midterm (part 1 of 2) / 2014.10.13 / MAT 2214.003 / Calculus III

Name: ______________________________________________

Please show all work and justify your answers.

1. Let $P$ be the plane in $\mathbf{R}^3$ spanned by $u = [1, -1, 0]$ and $v = [2, 1, 1]$. In other words, $P = \{su + tv\colon s, t \in \mathbf{R}\}$. In yet other words, $P$ is the unique plane containing $u$, $v$ and the origin. Find an equation for $P$ in terms of $x, y, z$. Sketch.

2. Let $T$ be the triangle whose vertices are the above points $u$, $v$ and the origin. In other words, $T = \{su + tv\colon 0 \leq s \leq 1 - t, 0 \leq t \leq 1\}$. What is the area of $T$? What are the lengths of sides of $T$? What are the angles of $T$?

3. Parametrize the line $L$ from $w = [-1, 0, 0]$ to the above plane $P$ that is perpendicular to $P$. For which value of your parameter does $L$ meet $P$? Find the point of intersection of $L$ and $P$. What is the distance from $w$ to the plane? Sketch.

| 1 | 2 | 3 | total (30) |
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4. Let $C$ be the curve in $\mathbf{R}^3$ given parametrically by $r(t) = [2\cos t, \sin t, \frac{1}{2}t]$.
   (a) Show that $C$ passes through the points $u = [\sqrt{2}, \frac{1}{2}\sqrt{2}, \frac{1}{8}\pi]$ and $v = [2, 0, 0]$.
   (b) Find a unit vector tangent to $C$ at $u$. Parametrize the line tangent to $C$ at $u$.
   (c) Express arclength along $C$ between $u$ and $v$ as a Calculus I integral. Sketch.
5. Let $p_1 = [1, 1, 2]$, $p_2 = [2, -1, 0]$. Parametrize the straight line segment $S$ from $p_1$ to $p_2$. Find the work done by the force field $F = [xy, y, -yz]$ in moving a particle along $S$.
6. Let $\omega = [2x + y]\,dx + [z\cos(yz) + x]\,dy + y\cos(yz)\,dz$.
   (a) Show $\omega$ is a closed form, i.e. $d\omega = 0$.
   (b) Show $\omega$ is exact by finding a scalar potential $\eta$ such that $d\eta = \omega$. Find all such $\eta$.
   (c) Find the integral of $\omega$ along any path from the origin to $[1, 2, 3]$.

| 4 | 5 | 6 | total (30) |
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