Final / 2014.12.16 / MAT 2214.003 / Calculus III

Name: ______________________________

Please show all work and justify your answers.

1. Parametrize the circle $x^2 + y^2 = a^2$ in the plane and find circulation of the vector field $F(x, y) = -y\,\widehat{\imath} + 2x\,\widehat{\jmath}$, both directly and using Green's theorem. Same for flux of $G(x, y) = -x\,\widehat{\imath} + 2y\,\widehat{\jmath}$ out of the circle.

2. Let $h > 0$. Parametrize the cone $x^2 + y^2 = z^2$, $0 \le z \le h$. Find its surface area. Sketch.

3. Find the plane tangent to the cone from the preceding problem at the point $[1, 1, \sqrt{2}]$, assuming $h > \sqrt{2}$. Find both unit normal vectors to the cone at that point. Sketch.

4. Let Ω be the triangular part of the plane $2x + y + z = 2$ cut out by the positive octant. Parametrize Ω using x and y as parameters. Use Kelvin-Stokes theorem to find circulation of the vector field $F = xz\,\widehat{\imath} + xy\,\widehat{\jmath} + 3xz\,\widehat{k}$ around the boundary triangle $\partial\Omega$. Sketch.

5. Let $a, h > 0$. Parametrize the cylinder $x^2 + y^2 = a^2, 0 \le z \le h$. Sketch. Find flux of the vector field G from problem 1 through the cylinder, oriented outward, both directly and closing up the cylinder and using Gauss-Ostrogradski theorem. Sketch.

6. Find all critical points of $f(x, y) = x^2 + y^2 - y + 2$ and classify them using the second derivative test.

7. Suppose the density of Ebola virus (in billions per square inch) in a Petri dish with a 3 inch diameter $x^2 + y^2 \le \frac{9}{4}$ is given by f from the preceding problem. At what coordinates is the density lowest? Highest?

8. Find the Hessian matrix of $f(x, y) = e^{xy}$. Find linear and quadratic Taylor approximations to f at $[1, 2]$. Compare values of your approximations to the "exact" values at four nearby points $[1 \pm 0.2, 2 \pm 0.2]$.

1	2	3	4	5	6	7	8	total (80)