Final exam / 2004.12.8 / Calculus III (honors) / MAT 2213.003

Name: _____

Please show all work and justify your statements. Label sketches, draw conclusions (using complete sentences and including units), and box your final answers as appropriate.

1. Determine whether $\dfrac{x^2y}{x^2+y^2}$ has a limit as $(x,y) \to (0,0)$.

   If yes, what is the limit? If no, explain why the limit fails to exist.

2. An airliner's heading is 5° north of east and its airspeed is 500 km/hr. If the airliner's progress over the ground is 520 km/hr due east, what is the velocity of the air current? You may ignore vertical components and treat this as a two dimensional problem.

3. What is the angle between the main diagonal of a cube and one of its edges?

4. Find the second order Taylor approximation to $x^y$ at the point $(2,1)$.

5. If you move north, relative humidity drops at the rate of 0.2 %/m and if you move southwest it rises at the rate of 0.1 %/m. Find the gradient of relative humidity at your location.

6. A conical pile of slush is melting in hot sun and spreading out under its own weight. The volume of the pile is one third the area of the base times the height. When the pile is 1 m high and has 3 m base diameter, its height is shrinking at the rate of 5 cm/min and its base diameter is spreading at the rate of 1 cm/min. How fast is the slush melting?

7. ACME roadrunner traps are made of wood and steel, each costing $p$ and $q$ dollars per unit respectively. The number of traps ACME can produce using $x$ units of wood and $y$ units of steel is $cx^ay^b$, where $a, b$, and $c$ are positive constants. If ACME's budget for raw materials is $B$ dollars, what is the largest number of traps they can produce?

8. A quonset hut is shaped like a half cylinder of radius 5 m and length 40 m. The hut is filled with hay, which is compressed under its own weight in such a way that the density varies linearly with height from 100 kg/m$^3$ at the top to 200 kg/m$^3$ at the bottom. Set up, but do not evaluate, an iterated integral for the total mass of hay in the hut. Sketch the hut and indicate your coordinate system in the sketch.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | total (80) | % |
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