Calculus III, MAT 2213-5
Final, May 10,1994
Instructor: D. Gokhman

Name: _____

1. (30 pts.) Let $\vec{u}=\vec{i}+2\vec{k}$, $\vec{v}=-2\vec{i}+\vec{j}$, $\vec{w}=3\vec{j}-\vec{k}$. Calculate the following:

   (a) $\vec{u}\cdot(\vec{v}+3\vec{w})$ (b) $\vec{u}\cdot(\vec{v}\times\vec{w})$ (c) $(\vec{u}\times\vec{v})\cdot\vec{u}$

   (d) $|v-w|$ (e) $\text{comp}_u v$ (f) $\text{proj}_u v$

2. (20 pts.) True/false questions. No explanation required.

T F (a) There is only one vector of length zero.

T F (b) If $\vec{u}\times\vec{v}=0$, then $u=0$ or $v=0$ or u is perpendicular to v.

T F (c) The line $\vec{r}\,(t)=t(\vec{i}+\vec{j}+\vec{k})$ is perpendicular to the plane $x+y+z=1$.

T F (d) The line $\vec{r}\,(t)=t\,\vec{i}$ lies in the plane $y+z=0$.

3. (40 pts.) Let $A=\vec{i}+2\vec{k}$, $B=-2\vec{i}+\vec{j}$, $C=3\vec{j}-\vec{k}$

   (a) Find a parametric formula for the line through A and B.

   (b) Find an equation for the plane through C perpendicular to the line.

   (c) Find the distance from A to the plane.

   (d) Find the distance from C to the line.

4. (40 pts.) Consider the plane curve $\vec{r}\,(t)=e^{t^2}\,\vec{i}+t^5\,\vec{j}$.

   (a) Find $r'(t)$ and $r''(t)$.

   (b) Find $r(1)$, $r'(1)$ and $r''(1)$.

   (c) Find a parametric formula for the line tangent to $\vec{r}\,(t)$ at the point $r(1)$.

   (d) Find a parametric formula for the line perpendicular to $\vec{r}\,(t)$ at the point $r(1)$.

5. (30 pts.) Consider the circle of radius 2 centered at $P=\vec{i}+\vec{j}$.

   (a) Find a parametric formula for the circle.

   (b) Find all x intercepts of the circle.

   (c) Pick one of the intercepts and find a parametric formula for the line tangent to the circle at that point.

1	2	3	4	5	6	7	8	total (240)

6. (20 pts.) Compute the limits of the following functions as $(x, y) \to (0, 0)$:

$$\text{(a) } x^2 + y^2 \qquad \text{(b) } xy \qquad \text{(c) } \frac{x}{x^2 + y^2} \qquad \text{(d) } \frac{xy}{x^2 + y^2}$$

7. (40 pts.) Let $f(x, y) = x^3 + y^3 + 5xy$.

   (a) Find all the first and second partial derivatives of f.

   (b) Find and classify all critical points of f.

   (c) Find the values of f at all critical points.

   (d) Sketch the traces by the planes $x = 0$ and $x - y = 0$.

8. (20 pts.) Let $f(x, y) = x^2 + y^2$.

   (a) Find an equation for the plane tangent to the graph of f at the point given by $x = 1$, $y = 1$.

   (b) Sketch the level curve going through this point.