Calculus III, MAT 2213-3
Final, May 12,1994
Instructor: D. Gokhman

Name: _____

1. (30 pts.) Let $\vec{u} = 2\,\vec{i} + \vec{k}$, $\vec{v} = \vec{i} - 2\,\vec{j}$, $\vec{w} = -\,\vec{j} + 5\,\vec{k}$. Calculate the following:

   (a) $\vec{u} \cdot (\vec{v} + 3\,\vec{w})$ (b) $\vec{u} \cdot (\vec{v} \times \vec{w})$ (c) $(\vec{u} \times \vec{v}) \cdot \vec{u}$

   (d) $|v - w|$ (e) $\operatorname{comp}_u w$ (f) $\operatorname{proj}_u w$

2. (20 pts.) True/false questions. No explanation required.

T F (a) There are exactly two unit vectors perpendicular to a plane in $\mathbf{R}^3$.

T F (b) If $\vec{u} \times \vec{v} = 0$, then $u = 0$ or $v = 0$ or u is perpendicular to v.

T F (c) The line $\vec{r}(t) = t(-\,\vec{i} - \vec{j} - \vec{k})$ is perpendicular to the plane $x + y + z = -2$.

T F (d) The line $\vec{r}(t) = t\,\vec{j}$ lies in the plane $x + z = 0$.

3. (40 pts.) Let $A = 2\,\vec{i} + \vec{k}$, $B = -\,\vec{i} + 2\,\vec{j}$, $C = \vec{j} - 3\,\vec{k}$

   (a) Find a parametric formula for the line through A and B.
   (b) Find an equation for the plane through C perpendicular to the line.
   (c) Find the distance from A to the plane.
   (d) Find the distance from C to the line.

4. (40 pts.) Consider the plane curve $\vec{r}(t) = t^2\,\vec{i} - e^{t^5}\,\vec{j}$.

   (a) Find $r'(t)$ and $r''(t)$.
   (b) Find $r(1)$, $r'(1)$ and $r''(1)$.
   (c) Find a parametric formula for the line tangent to $\vec{r}(t)$ at the point $r(1)$.
   (d) Find a parametric formula for the line perpendicular to $\vec{r}(t)$ at the point $r(1)$.

5. (30 pts.) Consider the circle of radius 2 centered at $P = \vec{i} - \vec{j}$.

   (a) Find a parametric formula for the circle.
   (b) Find all x intercepts of the circle.
   (c) Pick one of the intercepts and find a parametric formula for the line tangent to the circle at that point.

1	2	3	4	5	6	7	8	total (240)

6. (20 pts.) Compute the limits of the following functions as $(x, y) \to (0, 0)$:

$$\text{(a) } x^2 + y^2 \qquad \text{(b) } x^2 y^2 \qquad \text{(c) } \frac{xy}{x^2 + y^2} \qquad \text{(d) } \frac{x^2 y}{x^2 + y^2}$$

7. (40 pts.) Let $f(x, y) = x^3 + y^3 + 2xy$.

   (a) Find all the first and second partial derivatives of f.
   (b) Find and classify all critical points of f.
   (c) Find the values of f at all critical points.
   (d) Sketch the traces by the planes $x = 0$ and $x - y = 0$.

8. (20 pts.) Let $f(x, y) = x^2 + y^2$.

   (a) Find an equation for the plane tangent to the graph of f at the point given by $x = 1$, $y = -1$.
   (b) Sketch the level curve going through this point.