Calculus III, MAT 2213 Final Exam, May 4, 1993 Instructor: D. Gokhman
Show all pertinent work, answers alone are not sufficient. Box the answers.
Name: $\qquad$

1. (40 pts.) SEQUENCES AND SERIES. Determine whether each of the following sequences or series converges to a real number.
(a) $\frac{(\ln (n))^{5}}{n^{\frac{1}{n}}}$
(b) $\left(\frac{n}{n+1}\right)^{\left(n^{2}\right)}$
(c) $\sum_{n=1}^{\infty} \frac{4 n^{5}+4}{n^{7}+2 n^{3}}$
(d) $\sum_{n=1}^{\infty} \frac{n^{\frac{1}{2}} \cos (n \pi)}{\left(n^{5}+1\right)^{\frac{1}{4}}}$
2. (20 pts.) (a) Evaluate $\sum_{n=0}^{\infty} \frac{1}{2^{n}}+\frac{2}{3^{n}}$.
(b) Find the interval of convergence for the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 5^{n}}{n} x^{n}$.
3. (20 pts.) Find the Taylor series with center $c$ for the following functions

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\begin{array}{ll}
\text { (a) } f(x)=x e^{\frac{x}{5}}, \quad c=0 & \text { (b) } f(x)=\frac{1}{x^{2}}, \quad c=1
\end{array}
$$

4. (20 pts.) CURVES. Sketch the curves and find equations describing them.
(a) a circle passing through the origin with center $(-3,4)$.
(b) a parabola with vertex $(5,-2)$ and focus $(5,2)$.
5. (20 pts.) Suppose the position of a spider on the floor is given as a function of time $t$ by $x=t^{2}+2 t, y=t^{3}+3 t$. At time $t$ find the velocity components $\frac{d x}{d t}, \frac{d y}{d t}$ and the slope $\frac{d y}{d x}$. At time $t=0$ find an equation for the tangent line.
6. (40 pts.) POLAR COORDINATES.
(a) Sketch the curve $r=\cos (2 \theta)$. Find the area enclosed by one petal.
(b) Convert to Cartesian coordinates and sketch $r=2 \sec \left(\theta+\frac{\pi}{4}\right)$.
7. (40 pts.) VECTORS.
(a) Find the distance between the plane $x+4 y-z=-4$ and $(4,2,3)$.
(b) Find an equation for the plane containing the points $(-1,3,2)$, $(2,3,0),(-3,-1,0)$.
(Hint: pick 2 vectors parallel to the plane and take the cross product).

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | total (200) |
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