Calculus III, MAT 2213 Final Exam, May 4, 1993 Instructor: D. Gokhman Show all pertinent work, answers alone are not sufficient. Box the answers.

Name: _____

1. (40 pts.) SEQUENCES AND SERIES. Determine whether each of the following sequences or series converges to a real number.

(a)
$$\frac{(\ln(n))^5}{n^{\frac{1}{n}}}$$
 (b) $\left(\frac{n}{n+1}\right)^{\binom{n^2}{}}$ (c) $\sum_{n=1}^{\infty} \frac{4n^5+4}{n^7+2n^3}$ (d) $\sum_{n=1}^{\infty} \frac{n^{\frac{1}{2}}\cos(n\pi)}{(n^5+1)^{\frac{1}{4}}}$

- 2. (20 pts.) (a) Evaluate $\sum_{n=0}^{\infty} \frac{1}{2^n} + \frac{2}{3^n}$. (b) Find the interval of convergence for the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}5^n}{n} x^n$.
- 3. (20 pts.) Find the Taylor series with center c for the following functions

(a)
$$f(x) = xe^{\frac{x}{5}}$$
, $c = 0$ (b) $f(x) = \frac{1}{x^2}$, $c = 1$

- 4. (20 pts.) CURVES. Sketch the curves and find equations describing them.
 - (a) a circle passing through the origin with center (-3, 4).
 - (b) a parabola with vertex (5, -2) and focus (5, 2).
- 5. (20 pts.) Suppose the position of a spider on the floor is given as a function of time t by $x = t^2 + 2t$, $y = t^3 + 3t$. At time t find the velocity components $\frac{dx}{dt}$, $\frac{dy}{dt}$ and the slope $\frac{dy}{dx}$. At time t = 0 find an equation for the tangent line.
- 6. (40 pts.) POLAR COORDINATES.
 - (a) Sketch the curve $r = \cos(2\theta)$. Find the area enclosed by one petal.
 - (b) Convert to Cartesian coordinates and sketch $r = 2 \sec \left(\theta + \frac{\pi}{4}\right)$.
- 7. (40 pts.) VECTORS.
 - (a) Find the distance between the plane x + 4y z = -4 and (4, 2, 3).
 - (b) Find an equation for the plane containing the points (-1,3,2), (2,3,0), (-3,-1,0).
 (Hint: pick 2 vectors parallel to the plane and take the cross product)

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1	2	3	4	5	6	7	total (200)