Midterm 2 / 2022.10.27 / MAT 1313.001 / Algebra and number systems

Name: $\qquad$
Please show all work. Check your answers! $\because$

1. Find all integer solutions to the equation $48 x-34 y=6$
2. Solve the following system of two congruence equations

$$
\begin{gathered}
4 x \equiv 10 \bmod 13 \\
6 x \equiv 9 \bmod 11
\end{gathered}
$$

3. Suppose a sequence $x_{n}$ is defined recursively by $x_{0}=2, x_{1}=4$ and for $n>1$ $x_{n}=4 x_{n-1}-3 x_{n-2}$. Prove by induction that $x_{n}=3^{n}+1$ for all $n \geq 0$
4. Prove that $8^{\frac{1}{7}}=\sqrt[7]{8}$ is irrational.

| 1 | 2 | 3 | 4 | total (40) |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |

