

Name: \_\_\_\_\_

Please show all work.

1. (a) If  $P, Q, R$  are propositions, use a truth table to prove that  
$$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$$
  
(b) If  $X, Y, Z$  are sets, prove that  $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$
2. Using formal language and appropriate quantifiers, translate into symbolic form the following sentences. Determine whether they equivalent and explain why or why not.
  - Every integer is even or odd.
  - Every integer is even or every integer is odd.
3. For each statement below determine whether it is true. If so, prove it. If not, exhibit a concrete counterexample and explain why it is indeed a counterexample.
  - (a) If  $a, b, c$  are integers such that  $a$  divides  $b$  and  $b$  divides  $c$ , then  $a$  divides  $c$ .
  - (b) If  $a, b, c$  are integers such that  $a$  divides  $c$  and  $b$  divides  $c$ , then  $ab$  divides  $c$ .
4. For each statement below determine whether it is true. If so, prove it. If not, exhibit a concrete counterexample and explain why it is indeed a counterexample.
  - (a) If  $S$  and  $T$  are sets,  $S \cup T = S \Leftrightarrow S = T$
  - (b) If  $S$  and  $T$  are sets,  $S \cap T = S \Leftrightarrow S \subseteq T$

1	2	3	4	total (40)