Name: $\qquad$

Please show all work.

1. An integer $n$ is prime means $n>1$ and the only positive divisors of $n$ are 1 and itself.
(a) Write this definition in formal language using appropriate quantifiers.
(b) Negate the formal expression and simplify (show work)
(c) Write out the negation in words.
2. For each statement below determine whether it is true. If so, prove it (show work). If not, exhibit a concrete counterexample and explain why it is indeed a counterexample.
(a) If $S$ and $T$ are sets, $S \cup T=S \cap T \Leftrightarrow S \subseteq T$
(b) If $S$ and $T$ are sets, $S \cup T \subseteq S \cap T \Leftrightarrow S=T$
3. Consider the Diophantine equation $54 x-28 y=8$
(a) Use extended Euclid's algorithm to find the greatest common divisor of 54 and -28 and to find a certificate for it (show work)
(b) Find the general integer solution to the equation.
(c) Find three distinct particular integer solutions to the equation.

| 1 | 2 | 3 | total (30) |
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