Name: _____

Please show all work and justify your answers.

1. The fraction of the population of Moronica with annual income less than x thousand shekels is modeled by

$$0.004 \int_0^x t^2 e^{-.2t}\, dt$$

   (a) What are the mean (average) and median incomes in Moronica?

   (b) The value of the income distribution density at $x = 2$ is about 0.054. What does that say about incomes in Moronica?

2. Find the first order Fourier approximation to $f(x) = x$ on the interval $[-1, 1]$. Feel free to compute the required integrals numerically. Sketch x and the approximation over the entire interval on the same graph. What fraction of the total energy is captured by this approximation?

3. Suppose $y(x)$ is a solution of the differential equation

$$\frac{dy}{dx} = 2x(y^2 + 1)$$

   satisfying the initial condition $y(0) = 1$. Find $y(0.5)$ in two different ways.

   (a) Estimate using the forward Euler method with step size 0.25. Show all details.

   (b) Obtain an exact answer by finding the general family of solutions of the differential equation and selecting the solution satisfying the initial condition.

Fourier series: If f is a continuous function on $(-p/2, p/2)$, then $f(x) = a_0 + \sum_{k=1}^{\infty} [a_k \cos(2\pi kx/p) + b_k \sin(2\pi kx/p)]$,

where $a_0 = \frac{1}{p} \int_{-\frac{p}{2}}^{\frac{p}{2}} f(x)\, dx$, $a_k = \frac{2}{p} \int_{-\frac{p}{2}}^{\frac{p}{2}} f(x) \cos(2\pi kx/p)\, dx$, $b_k = \frac{2}{p} \int_{-\frac{p}{2}}^{\frac{p}{2}} f(x) \sin(2\pi kx/p)\, dx$

Energy theorem: $E = \frac{2}{p} \int_{-\frac{p}{2}}^{\frac{p}{2}} f(x)^2\, dx = \sum_{k=0}^{\infty} E_k$, where $E_0 = 2a_0^2$ and $E_k = a_k^2 + b_k^2$ for $k \geq 1$.

1	2	3	total (30)
			%

Prelim. course grade: %