Name:
Please show all work and justify your answers. Name the results you use, including integration formulas from the table. If you compute integrals numerically, reveal the details. Make and label sketches. Supply brief narration with your solutions and draw conclusions, including units as appropriate.

1. A wine glass has the shape of a paraboloid - a surface obtained by revolving the curve $y=1.5 x^{2}$ around the $y$ axis. If the diameter of the top surface is 8.2 cm , how much wine is there in the glass?

2. Determine whether the improper integral $\int_{1}^{\infty} \frac{\sqrt{\sqrt{x}+1}}{\sqrt[3]{x^{4}}} d x$ converges. Justify.
3. Bert and Ernie are given a definite integral. Based on their trapezoidal approximations 5.432 (Bert with 10 subdivisions) and 5.131 (Ernie with 15 subdivisions) estimate the exact value of the integral.
4. Demonstrate your mastery of techniques of integration (other than guess-and-check) by evaluating the following integrals. Name the techniques you are using.
(a) $\int x^{2} \cos x d x$
(b) $\int \frac{x^{2}}{x^{2}+2 x+1} d x$

| 1 | 2 | 3 | 4 | total (40) |
| :--- | :--- | :--- | :--- | :---: |
|  |  |  |  |  |
|  |  | Prelim. course grade: $\quad \%$ |  |  |

