

Name: _____

Please show all work and justify your answers. Name the results you use, including integration formulas from the table. When you compute integrals numerically, reveal the details. Make and label sketches. Supply brief narration with your solutions and draw conclusions, including units as appropriate.

- The tip of the Rodong 1 missile is a solid of revolution obtained by rotating around the y axis the region in the plane bounded by $y = x^2$, $0 \leq y \leq 9$. If the mass density is given by $\delta = 2 - 0.1y$, (a) find the total mass of the tip, and (b) locate its center of mass.

- Determine whether the improper integral $\int_0^1 \sqrt{\frac{x+1}{x^3}} dx$ converges. Justify.

- Itchy and Scratchy are given a definite integral. Based on their Simpson approximations 6.274 (Itchy with 10 subdivisions) and 6.183 (Scratchy with 25 subdivisions) estimate the exact value of the integral.

- Demonstrate your mastery of techniques of integration by evaluating the following integrals. Show all steps and name the techniques you are using.

$$(a) \int x^2 \ln x dx \quad (b) \int \frac{x^2}{x^2 - 2x + 1} dx$$

- Let P denote the probability of a Gaggme espresso machine breaking t months after purchase. Experiments suggest that $P'(t)$ is proportional to a decaying exponential $e^{-0.05t}$.

- What is the probability density $P'(t)$? (i.e. what is the proportionality constant?)
- What is the probability of breakage during the 2nd month after purchase?
- Find the mean and the median of the distribution.

- Find the third order Taylor approximation to $\cos(x)/(1+x)$ at $x = 0$. Sketch the given function and the approximation near $x = 0$ on the same graph.

- Find the first order Fourier approximation to $f(x) = 2 - |x|$ on the interval $[-2, 2]$. Sketch x and the approximation over the entire interval on the same graph. What fraction of the total energy is captured by this approximation?

- Suppose $y(x)$ is a solution of the differential equation $y' = 3(yx + x)^2$ satisfying the initial condition $y(1) = -2$. Find $y(1.5)$ in two different ways.

- Estimate using the forward Euler method with step size 0.25. Show all details.
- Obtain an exact answer by finding the general family of solutions of the differential equation and selecting the solution satisfying the initial condition.

Hint: factor.

1	2	3	4	5	6	7	8	total (80)
								%