

Name: \_\_\_\_\_

Please show all work and justify your statements. Make and label sketches, draw conclusions (using complete sentences and including units), and box the final answers as appropriate.

1. Evaluate  $\lim_{x \rightarrow 0} f(x)$ . Find  $a$  that makes  $f$  continuous or explain why no  $a$  works.

$$(a) f(x) = \begin{cases} \frac{|x|}{x} & \text{for } x \neq 0 \\ a & \text{for } x = 0 \end{cases} \quad (b) f(x) = \begin{cases} \frac{\sin x}{x} & \text{for } x \neq 0 \\ a & \text{for } x = 0 \end{cases}$$

$$(c) f(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ a & \text{for } x = 0 \end{cases} \quad (d) f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ a & \text{for } x = 0 \end{cases}$$

2. Use the definition of the derivative to find the equation of the line tangent to the graph of  $y = x^{-1}$  at the point  $[2, \frac{1}{2}]$ . Sketch.
3. Find derivatives of the following functions with respect to  $x$ .

$$(a) \sqrt{x} \ln x \quad (b) \frac{x}{1+x^2} \quad (c) \cos(e^x) \quad (d) \sin(x^e)$$

4. Suppose  $f$  is a differentiable function. Apply the rules of differentiation to the functions  $f(x^2)$  and  $x^2 f(x)$  and explain why both their slopes at  $x = 0$  must be 0.
5. Find two lines through the point  $[-3, 8]$  that are tangent to the graph of  $y = x^2$ .

1	2	3	4	5	total (50)