

Name: _____

Please show all work and justify your statements. Make and label sketches, draw conclusions (using complete sentences and including units), and box the final answers as appropriate.

1. Betty jumps off the roof of a skyscraper. Part of the way down she opens her parachute and glides down to the ground. Sketch Betty's altitude and speed as functions of time from slightly before she jumps to slightly after she lands. Are these functions continuous? Explain.

2. Evaluate the following limits. Justify your answers.

(a) $\lim_{x \rightarrow \infty} \frac{1-x}{1-x^3}$ (b) $\lim_{x \rightarrow 1} \frac{1-x}{1-x^3}$

3. Let $f(x) = \frac{1}{x^2}$.

- (a) Use the definition of derivative to find f' and show that it satisfies the power rule.
- (b) Find an equation for the tangent line to f at $x = 1$. Sketch.

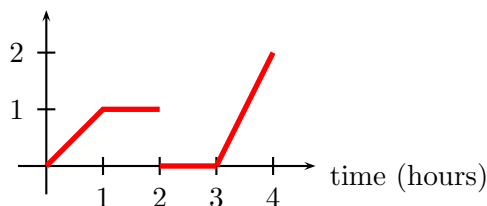
4. On what intervals is the graph of $y = \frac{1}{1+x^2}$ concave down?

5. Find dy/dx , where $\pi^y \arctan(2x) = \cosh(x^2y)$.

6. Market research shows that if a piece of software is priced at p dollars (for a certain range of p), then $q = 1500 - 50p$ will be the quantity sold. Use calculus to explain how the software should be priced to maximize revenue?

7. A submarine springs a leak and the sailors battle to plug it up. The rate of intake of water is graphed below. On a separate graph make an accurate sketch of the quantity of water inside the submarine as a function of time. How much water did the submarine take on after 4 hours?

rate of intake (100 gallons/hour)



8. Evaluate the following integrals analytically. Show all steps.

(a) $\int_1^4 \frac{1}{t^2\sqrt{t}} dt$ (b) $\int_{-\pi/2}^{\pi} \sin(7t) dt$ (c) $\int \frac{1+t}{t} dt$ (d) $\int \pi^{et} dt$

9. A medical examination reveals a tumor 2 mm in diameter, which grows at a rate proportional to the square root of time. After a month the tumor grows to 3 mm. How large will the tumor be a year after the initial diagnosis?

1	2	3	4	5	6	7	8	9	total (90)