Name:
Please show all work and justify your statements. Make and label sketches, draw conclusions (using complete sentences and including units), and box the final answers as appropriate.

1. (10 pts.) In a two-dimensional universe far way, a spaceship leaves its home planet at the origin and flies along a trajectory given as a function of time by $x(t)=2^{t}-1$, $y(t)=\sin (\pi t)$. When the ship reaches the coordinates $x=1, y=0$ the crew mutinees and releases the captain in an escape pod. In accordance with Newton's law of inertia, due to the absense of significant gravitational sources nearby, the pod continues flying roughly in a straight line with constant speed.
(a) Sketch the trajectory of the spaceship.
(b) When does the mutiny occur?
(c) What are the escape pod's coordinates as functions of time after the mutiny?
(d) Sketch the pod's trajectory on your previous drawing. Does it look plausible?
(e) What is the escape pod's speed?
2. (10 pts.) An enterpreneur is planning to develop and market a new video game. Preliminary research indicates that pricing each unit at $\$ 50$ will result in 100,000 sales over the lifetime of the product. In addition, raising the price would cause a drop in sales by 1,000 for each $\$ 1$ increase, while cutting the price would increase sales by 1,000 for each $\$ 1$ drop in price. If it costs $\$ 1$ to deliver each unit, how should the game be priced to maximize profit? What is the upper bound on acceptable development costs for the game?
3. (10 pts.) A model rocket with a sputtering engine is launched straight up. A built-in speedometer measures the upward velocity of the rocket. The graph below shows velocity as a function of time. On the same set of axes sketch the graph of the rocket's position as a function of time. Indicate the critical points on the graph of position.

4. (10 pts.) Evaluate the following integrals. Show all steps.
(a) $\int_{1}^{4} t \sqrt{t^{3}} d t$
(b) $\int_{-\frac{\pi}{2}}^{\pi} \cos (3 t) d t$
(c) $\int\left[\frac{1-t}{t}\right]^{2} d t$
(d) $\int 3^{5 t} d t$
5. (10 pts.) Black mold starts growing on the walls of Chisholm hall. Its rate of growth is roughly proportional to the cube root of time. After a week, the mold forms a 1 mm thick layer. How much mold do we expect 20 weeks after the initial infestation?

