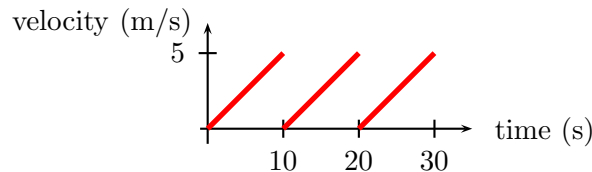
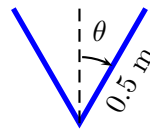


Name: _____

Please show all work and justify your statements. Make and label sketches, draw conclusions (using complete sentences and including units), and box the final answers as appropriate.

- The population of a country as a function of time is modeled by $P(t) = a(1 + b)^t$, where a and b are positive constants. Is the rate of population growth increasing or decreasing? Explain using calculus. Hint: $\ln x > 0$, if $x > 1$.
- Let $f(x) = \sqrt{2x}$.
 - Use the definition of derivative to find f' and show that it satisfies the power rule.
 - Find an equation for the tangent line to f at $x = 2$. Sketch.
- On what intervals is the graph of $y = \arctan(x^2)$ concave up? Hint: $(\arctan x)' = \frac{1}{1+x^2}$.
- Find dy/dx , if (a) $y^e + e^x = \pi$ (b) $xy = \ln(x^2 + y^2)$
- Farmer Bob builds a trough for his pigs by folding lengthways a piece of sheet metal 5 m long and 1 m wide. The cross section of the trough is shown in the figure. Assume that the angle θ is 30 degrees. If water enters the trough at the rate of half a liter per second, how fast is the water level rising when the trough is filled half way up? Hint: A liter is 10^{-3} m^3 .
- A spectator throws a roach onto a basketball court. At time $t = 0$ the roach lands and starts crawling. Its subsequent position is given by $x(t) = \cos(\pi t)$, $y(t) = 3^t - 1$.
 - Where does the roach land? Sketch the roach's path.
 - At what time does the roach reach the point $(1, 8)$?
 - Give a parametric formula for the line tangent to the roach's path at $(1, 8)$. Sketch.
- Farmer Bob wants to maximize the volume of his trough by changing θ . Help him out.
- A walking inebriate speeds up, hits a lamp post, speeds up again, hits another lamp post, and so on. His speed is given by the graph below. On the same set of axes, carefully sketch his position as a function of time. How far apart are the lamp posts?



9. Evaluate the following integrals. Show all steps.

(a) $\int_1^2 \frac{1}{\sqrt{t^3} \sqrt[3]{t^2}} dt$ (b) $\int_{-\pi/2}^{\pi} \cos(5t) dt$ (c) $\int \sqrt{t+1} dt$ (d) $\int \pi^{et} dt$

10. During a check-up an x-ray shows a gallstone with diameter 1 mm. A month later the diameter has increased to 1.2 mm. If the rate of gallstone's growth is proportional to the square root of time, what will be the size of the gallstone a year after the initial checkup?

1	2	3	4	5	6	7	8	9	10	total (100)