Name: _____

Please show all work and justify your answers.

1. (20 pts.) Evaluate each of the following limits or state that the limit does not exist.

   (a) $\lim_{x\to -2} \dfrac{x^2-4}{x^3+8}$ (b) $\lim_{x\to -2^-} \dfrac{2+x}{|2+x|}$ (c) $\lim_{t\to 0} \dfrac{\sin(2t)}{\tan(3t)\cos(t^3)}$ (d) $\lim_{t\to 0} t^2 \sin(1/t^3)$

2. (20 pts.) Find the derivatives of the following functions

   (a) $f(x) = \sin^2(\sqrt{x})$ (b) $f(x) = \dfrac{1}{\sqrt{2x^5}}$ (c) $f(x) = \dfrac{2x+1}{3x^2-5}$ (d) $f(x) = 3x^5\cos(x)$

3. (20 pts.) State the definition of derivative and use it (without resorting to rules of differentiation) to find $f'(4)$, where $f(x) = 1/\sqrt{x}$.

4. (20 pts.)

   (a) Find dy, where $\cos(x^3y - 1) = y$.

   (b) Find the linear approximation to $f(x) = \sqrt{x^2 - x + 2}$ near $x = 2$.
   Sketch both $y = f(x)$ and the linear approximation.

5. (20 pts.) Harold places a 5 m ladder against a building and climbs to the top without having noticed that he placed one leg of the ladder on a banana peel. The ladder starts slipping away and the bottom of the ladder moves with speed 0.2 m/sec when it is 0.5 m away from the building. How fast is Harold falling at that moment?

6. (25 pts.) Evaluate the following:

   (a) $\displaystyle\int_2^3 (x^3+1)\,dx$ (b) $\displaystyle\int_0^2 \sqrt{3x+1}\,dx$ (c) $\displaystyle\int_0^4 |x-1|^5\,dx$

   (d) $\dfrac{d}{dx}\displaystyle\int_2^x \sqrt{3+\cos(t)}\,dt$ (e) $\dfrac{d}{dx}\displaystyle\int_{x^3}^{x^2} \sqrt{2+\sin(t)}\,dt$

7. (20 pts.) Betty wants to construct a 80 m^2 rectangular enclosure next to a long wall. She needs to buy some fence for three new walls of the enclosure. What should the dimensions of the enclosure be in order to minimize the cost of fencing?

8. (20 pts.) Solve the differential equation $dy/dx = (2y+1)^2(3x+1)$ subject to the initial condition: $y = 2$ when $x = 0$. Sketch the solution.

1	2	3	4	5	6	7	8	total (165)	(%)