

Name: _____ Pseudonym: _____

Please show all work and box the answers.

1. (30 pts.) Evaluate each limit or state that it does not exist, and briefly explain why.

(a) $\lim_{x \rightarrow 0} \frac{x^2}{|x|}$ (b) $\lim_{x \rightarrow 1} \frac{\cos(x-1)}{x^3-1}$ (c) $\lim_{x \rightarrow 0} \frac{x}{\sin(2x)}$

2. (20 pts.) Without using the rules of differentiation, find $f'(x)$, where

(a) $f(x) = x^3$ (b) $f(x) = \frac{1}{x}$

3. (30 pts.) Find the derivatives of the following functions:

(a) $f(x) = x^3 + 3x^2 - x - 1$ (b) $f(x) = x \sin x$ (c) $f(x) = \cos^2 x^3$

4. (20 pts.) Find the equation of the tangent line to the graph of $y = \sqrt{(2x)}$ at $x = 2$. Sketch.

5. (20 pts.) Let $f(x) = \begin{cases} 2 & \text{for } x \leq -1 \\ mx + b & \text{for } -1 < x < 1 \\ -2 & \text{for } x \geq 1 \end{cases}$

For which values of m and b is $f(x)$ continuous? Sketch.

Is $f(x)$ differentiable? Explain.

6. (40 pts.) Let $f(x) = 2x^2 - x^4$.

(a) Find all critical points of f and state whether each is a local minimum, local maximum or neither.

(b) Sketch the graph of $y = f(x)$.

(c) Find the intervals where f is increasing, decreasing, concave up, concave down.

(d) Find the minimum and maximum values of f on the interval $[-2, 2]$.

7. (20 pts.) Solve for $y(x)$ the differential equation $\frac{dy}{dx} = \frac{1}{y^2}$, subject to the following initial conditions:

(a) $y(0) = 1$, (b) $y(0) = -1$.

8. (20 pts.) Evaluate the following definite integrals:

(a) $\int_{-1}^1 (x^4 - 1) dx$ (b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x) dx$

1	2	3	4	5	6	7	8	total (200)	%