

Name: _____

Please show all work. Supply brief narration and illustrations with your solutions and draw conclusions.

1. Find all critical points of $f(x) = x - x^3$ in the interval $-2 \leq x \leq 2$. Use f'' to determine whether they are local minima or maxima. Find the global minimum and maximum of f of the interval and state where they occur. Sketch.
2. Find indefinite integrals of the following functions

$$(a) \frac{e^{-2x}}{(1 + e^{-2x})^2} \quad (b) t^2 \cos(3t)$$

3. Determine whether the improper integral $\int_1^\infty \frac{dx}{x^{\frac{2}{3}} + x^{\frac{4}{3}}}$ converges or diverges. Justify your assertion by comparison to an integral whose convergence or divergence can be determined directly.
4. For the autonomous differential equation $dx/dt = x - a^3x^4$, where a is a positive constant, draw the phase-line diagram, find the equilibria, and determine their stability.
5. Solve the Torricelli differential equation $dh/dt = -\sqrt{h}$ with initial condition $h(0) = 5$. Sketch the solution and describe its long-term behavior.

1	2	3	4	5	total (50)

Prelim. course grade: %