

Name: \_\_\_\_\_

Please show all work. Supply brief narration with your solutions and draw conclusions.

1. The force between two atoms as a function of the distance  $x > 0$  between the atoms is given by  $-a/x^2 + b/x^3$ , where  $a$  and  $b$  are positive constants. Which value of  $x$  minimizes the force?
2. Find indefinite integrals of the following functions

$$(a) \frac{e^{-x}}{2 + e^{-x}} \quad (b) \frac{t}{e^t}$$

3. Determine whether the improper integral  $\int_0^1 \frac{1}{\sqrt{x} + x^2} dx$  converges or diverges. Justify your assertion by comparison to an integral whose convergence or divergence can be determined directly.
4. For the autonomous differential equation  $dx/dt = ax - x^3$ , where  $a$  is a positive constant, draw the phase-line diagram, find the equilibria, and determine their stability.
5. Solve the differential equation  $dh/dt = -h^2$  with initial condition  $h(0) = 2$ . Sketch the solution and describe its long-term behavior.

1	2	3	4	5	total (50)