

Name: \_\_\_\_\_

Please show all work. Supply brief narration with your solutions and draw conclusions.

1. Let  $f(t) = t^4 - 2t^2$ . Find all the critical points of  $f$  on the interval  $-2 \leq x \leq 2$ . Use the second derivative to determine concavity at the critical points. Find the global minimum and the global maximum of  $f$  on the interval. Where do they occur?
2. Find indefinite integrals of the following functions

(a)  $e^{2t}(1 + e^{2t})^5$       (b)  $t \cos(2t)$

3. Show that the improper integral  $\int_1^\infty \frac{1}{\sqrt{x} + x^2} dx$  converges and find an upper bound.
4. For the autonomous differential equation  $dx/dt = x - ax^2$ , where  $a$  is a positive constant, draw the phase-line diagram, find the equilibria, and determine their stability.
5. Solve the Torricelli equation  $dh/dt = -\sqrt{h}$  with initial condition  $h(0) = 1$ . When is  $h = 0$ ?

| 1 | 2 | 3 | 4 | 5 | total (50) | % |
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Prelim. course grade:      %