Final exam / 2008.12.10 / Calculus for the Biosciences / MAT 1194.002

Name: ______________________________

Please show all work. Supply brief narration with your solutions and draw conclusions.

1. An exponentially growing yeast culture doubles in 5 days starting from 2 million *saccaromyces cerevisiæ*. How long would it take it to reach a population of 10 million?

2. During treatment viral load decreases exponentially according to $v(t) = e^{-t}$. Find and illustrate on a graph

   (a) Load at $t = 0$ and $t = 1$.
   (b) The average rate of change between $t = 0$ and $t = 1$.
   (c) The instantaneous rates of change at $t = 0$ and $t = 1$.

3. Find the derivatives of

$$\text{(a) } e^{1+\cos(2x)} \qquad \text{(b) } \frac{\ln(\ln x)}{x}$$

4. Find the second derivative of $x/(1 + x^2)$ and use it to describe the curvature of the function's graph.

5. For the Ricker model for fish population $x_{t+1} = rx_te^{-x_t}$ find the equilibria. For which values of r is each equilibrium stable? Unstable?

6. Let $f(t) = t^2 - t^3$. Find all the critical points of f on the interval $0 \le x \le 2$. Use the second derivative to determine concavity at the critical points. Find the global minimum and the global maximum of f on the interval. Where do they occur?

7. Find indefinite integrals of the following functions

$$\text{(a) } \sin(3t)[1 + \cos(3t)]^7 \qquad \text{(b) } t\sin(5t)$$

8. Show that the improper integral $\displaystyle\int_1^\infty \frac{1}{x - \sqrt{x}}\,dx$ diverges.

9. For the autonomous differential equation $dx/dt = ax - x^3$, where a is a constant, draw the phase-line diagram, find the equilibria, and determine their stability. What happens if $a = 0$?

10. Solve the Torricelli equation $dh/dt = -\sqrt{2h}$ with initial condition $h(0) = 3$. When is $h = 0$?

1	2	3	4	5	6	7	8	9	10	total (100)