

Name: \_\_\_\_\_

Please show all work. Supply brief narration with your solutions and draw conclusions.

1. Find antiderivatives for the following functions

$$(a) \frac{t}{t^2 + 1} \quad (b) \frac{\ln t}{t} \quad (c) (t + 1) \cos(3t)$$

2. Let  $t$  be the number of seconds it takes for a *belieber* to cry after Justin takes the stage. Suppose the probability density of the onset of tears of joy is linear on the interval  $0 \leq t \leq 50$  given by  $p(t) = c(1 - 0.02t)$ , where  $c$  is a positive constant. Assume  $p(t) = 0$  for  $t$  outside the interval.

- (a) Sketch  $p(t)$  on the interval  $-10 \leq t \leq 60$ . What must be the value of  $c$ ?
- (b) What is the chance a random pair of eyes in the audience will stay dry for 30 seconds?
- (c) On average how long does it take for a fan to start crying?
- (d) At what time are half the fans at the show in tears?

3. Emotionally exhausted Justin is put on diazepam administered continuously at the rate of 6 milligrams per day. Diazepam is eliminated at the rate of 35% per day.

- (a) Write down a differential equation satisfied by the level  $y(t)$  of diazepam on day  $t$ .
- (b) Find equilibria, if any.
- (c) Sketch a slope field and determine stability of equilibria.
- (d) Find a solution with initial condition  $y(0) = 0$  and sketch it on the same graph.

4. Suppose  $y$  satisfies the differential equation  $\frac{dy}{dt} = \frac{y}{t + 1}$

- (a) Solve the equation with initial condition  $y(0) = 5$ .
- (b) Sketch the solution and describe in words its long-term behavior.

1	2	3	4	total (40)