

Name: _____

Please show all work. Supply brief narration with your solutions and draw conclusions.

- A researcher starts a bacterial culture in a petri dish. Three days later the colony is 5 million strong. The day after that it reaches 7 million. Assuming the growth is exponential, what will the size be on the fifth day?
- The level of medication for a while varies according to $s(t) = 15 + t^2 - t$ where time t is measured in days. Compute the derivative of s with the respect to t using the definition of derivative. Find and illustrate on a graph
 - Initial level and after 3 days.
 - The instantaneous rates of change at those two times.
 - The average rate of change during that period of time.
 - The equation of the tangent line at $t = 3$.
- A population of wasps x_t satisfies the recursion $x_{t+1} = 5\sqrt{x_t}$. Find fixed points of the recursion (equilibria) and do some cobwebbing on a graph or numerical experimentation to determine their stability (attracting vs. repelling). Describe what happens to the population in the long run, if $x_0 = 0$. Same, if $x_0 = 1$.
- Find y' where

$$(a) y = x^4 \ln x \quad (b) y = \frac{x^3}{\sin(2x)} \quad (c) y = x^{e^x} \quad (d) \sin(2x) + \exp(3y) = y^2$$

- The windpipe contracts during a cough from the rest radius R to radius $r \leq R$. The speed of the exiting is $v = a(R - r)r^2$, where a is a positive constant. What value of r maximizes speed?
- Evaluate the following limits. Justify your answers. If you use l'Hôpital's rule, be sure to specify which case.

$$(a) \lim_{x \rightarrow \infty} \frac{x}{3x + 1} \quad (b) \lim_{x \rightarrow 0} x^4 \sin\left(\frac{1}{x}\right) \quad (c) \lim_{x \rightarrow 0} \frac{\sin(3x)}{5x} \quad (d) \lim_{x \rightarrow 0^+} x^4 \ln x$$

- Find antiderivatives for the following functions.

$$(a) \sin(3x) \quad (b) x \sin(3x^2) \quad (c) x^2 \sin(3x) \quad (d) \ln x \text{ (hint: by parts)}$$

- After an initial bolus injection of 22 mg of a drug, a patient is placed on a drip delivering 1.2 mg per hour. If the drug is cleared by the patient at a rate of $0.8 - 0.05t^3$ mg/h as a function of time, what is the amount of drug in the patient 4 hours later?

1	2	3	4	5	6	7	8	total (80)	%