Final exam / 2011.12.14 / Calculus for the Biosciences / MAT 1193.005

Name: ____________________

Please show all work. Supply brief narration with your solutions and draw conclusions.

1. A researcher starts a bacterial culture in a petri dish. Three days later the colony is 25 million strong. The next day it reaches 30 million. Assuming the growth is exponential, what was the initial size?

2. Find the derivatives of

$$\text{(a) } t^2 7^{3^t} \qquad \text{(b) } \frac{\ln t}{\sqrt{2t^3}} \qquad \text{(c) } \sin t \, \cos t$$

3. Find the second derivative of $f(t) = \dfrac{1}{t^2+1}$ and use it to describe the curvature of the graph of f for $t \geq 0$.

4. A population x_t has *per capita* production $\dfrac{2}{x_t+1}$. Write down the discrete dynamical system for x_t. Find equilibria and do some cobwebbing on a graph to determine their stability. Find the derivative of the updating function. What are its values at each equilibrium? Describe in words what happens in the long run.

5. Find all critical points of $\sin x$ in the interval $0 \leq x \leq 2\pi$. Use f'' to determine whether they are local minima or maxima. Find the global minimum and maximum of f of the interval and state where they occur. Sketch.

6. Find indefinite integrals of the following functions

$$\text{(a) } \frac{e^{3x}}{(1-5e^{3x})^2} \qquad \text{(b) } \frac{1}{x(\ln x)^2} \qquad \text{(c) } t^3\, e^{-5t}$$

7. Determine whether the improper integral $\displaystyle\int_1^\infty \frac{dx}{x^{\frac{4}{5}} + x^{\frac{6}{5}}}$ converges or diverges. Justify your assertion by comparison to an integral whose convergence or divergence can be determined directly.

8. For the autonomous differential equation $dx/dt = x - ax^2$, where a is a positive constant, draw the phase-line diagram, find the equilibria, and determine their stability.

9. Solve the differential equation $dh/dt = -h^2$ with initial condition $h(0) = 2$. Sketch the solution and describe its long-term behavior.

1	2	3	4	5	6	7	8	9	total (90)	%