

Name: \_\_\_\_\_

Please show all work and justify your answers.

1. Let  $m \geq 2$ . Prove that if  $a \equiv b \pmod{m}$ , then  $\gcd(a, m) = \gcd(b, m)$ . Does the converse hold? Explain.
2. Apply the extended Euclidean algorithm to find  $\gcd(244, 224)$  and the Bézout coefficients.
3. Use the Chinese remainder formula to solve the following system of congruences:

$$x \equiv 1 \pmod{7}, \quad x \equiv 2 \pmod{8}, \quad x \equiv 3 \pmod{9}.$$

4. Prove by induction that  $2^n \geq 1 + n$  for all  $n \geq 1$ .

1	2	3	4	total (40)