Midterm 1 / 2016.2.19 / CS 3333.002 / Mathematical foundations of computer science

Name: $\qquad$
Please show all work and justify your answers.

1. Let $m \geq 2$. Prove that if $a \equiv b \bmod m$, then $\operatorname{gcd}(a, m)=\operatorname{gcd}(b, m)$. Does the converse hold? Explain.
2. Apply the extended Euclidean algorithm to find $\operatorname{gcd}(244,224)$ and the Bézout coefficients.
3. Use the Chinese remainder formula to solve the following system of congruences:

$$
x \equiv 1 \bmod 7, \quad x \equiv 2 \bmod 8, \quad x \equiv 3 \bmod 9
$$

4. Prove by induction that $2^{n} \geq 1+n$ for all $n \geq 1$.

| 1 | 2 | 3 | 4 | total (40) |
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