Midterm 1 / 2016.2.19 / CS 3333.002 / Mathematical foundations of computer science

## Name: \_

Please show all work and justify your answers.

- 1. Let  $m \ge 2$ . Prove that if  $a \equiv b \mod m$ , then gcd(a, m) = gcd(b, m). Does the converse hold? Explain.
- 2. Apply the extended Euclidean algorithm to find gcd(244, 224) and the Bézout coefficients.
- 3. Use the Chinese remainder formula to solve the following system of congruences:

 $x \equiv 1 \mod 7, \qquad x \equiv 2 \mod 8, \qquad x \equiv 3 \mod 9.$ 

4. Prove by induction that  $2^n \ge 1 + n$  for all  $n \ge 1$ .

1	2	3	4	total (40)