Name: _____

Please show all work and justify your answers.

1. How many permutations of the 10 decimal digits contain the string 42?

2. What's more likely, rolling 8 with 5 dice or 7 with 3?

   Hint: k-combinations out of n with repeats: $C(n+k-1, k)$, where $C(r, k) = P(r, k)/k!$.

3. Suppose E and F are independent events, i.e. $P(E \cap F) = p(E)p(F)$. Show that $\overline{E}$ and $\overline{F}$ are independent.

   Hints: Use de Morgan's law $\overline{E} \cap \overline{F} = \overline{E \cup F}$ and the inclusion-exclusion principle $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

4. Let t be the number of days it takes for one of Justin Bieber's tattoos to heal. Assume that $c > 0$ and the probability density function for t is

$$p(t) = \begin{cases} c\,(1 - 0.02\,t) & \text{for } 0 \le t \le 50, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Sketch $p(t)$ on the interval $-10 \le t \le 60$. What value of c makes $p(t)$ into a probability density? Use this value of c to answer the following questions.

   (b) What is the likelihood that a random tattoo will heal within 10 days?

   (c) How long does it take on average for Justin's tattoos to heal?

   (d) If Justin goes wild and gets lots of tattoos at once, how many days will it take for half of them to heal?

   Hint: Find the median of $p(t)$, i.e. a number M such that the vertical line $t = M$ splits the area under $p(t)$ exactly in half.

1	2	3	4	total (40)