Midterm 2 / 2014.3.28 / CS 3333.001 / Mathematical foundations of computer science

Name: _____

Please show all work and justify your answers.

1. Consider the linear system $x + y = 3$, $z = y + 2x$.
   (a) What augmented matrix A represents this system? Use Gauss-Jordan elimination to find its reduced row echelon form. Show steps.
   (b) Use (a) to find all solutions to the system in terms of the free variable(s). Check your answer.

2. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.
   (a) Compute the determinant of A. Show steps. What does your answer say about the effect on volumes by the linear transformation of Euclidean space given by $x \mapsto Ax$?
   (b) Compute A^{-1}. Show steps and check your answer.
   (c) Use (b) to solve the linear system $AX = B$ for the matrix X.

3. Let $A = \begin{bmatrix} -2 & 3 \\ 6 & 1 \end{bmatrix}$.
   (a) Find the characteristic polynomial of A and its roots — the eigenvalues of A.
   (b) For each of the eigenvalues you found in (a) find corresponding eigenvectors.
   (c) Find an invertible matrix P such that $P^{-1}AP$ is diagonal. Check your answer.
   (d) Sketch the eigenspaces and describe the geometrical effects of the plane transformation $x \mapsto Ax$.

1	2	3	total (30)